

# 數數

數的故事

egg

Sep. 16th, 2008 (1st Edit)

Sep. 30th, 2008 (2nd Edit)

# 數怎麼數?

▶ skip

# 數怎麼數?

1,

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# 數怎麼數?

1, 2,

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# 數怎麼數?

1, 2, 3,

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# 數怎麼數?

1, 2, 3, 4,

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# 數怎麼數?

1, 2, 3, 4, 5,

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# 數怎麼數？

1, 2, 3, 4, 5, 6,

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# 數怎麼數?

1, 2, 3, 4, 5, 6, 7,

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# 數怎麼數?

1, 2, 3, 4, 5, 6, 7, 8,

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# 數怎麼數?

1, 2, 3, 4, 5, 6, 7, 8, 9,

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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16, 17,

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# 數怎麼數?

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16, 17, 18,

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數？

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數？

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,  
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# 數怎麼數?

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51, 52, 53, 54, 55,

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# 數怎麼數?

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,  
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51, 52, 53, 54, 55, 56,

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# 數怎麼數?

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,  
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# 數怎麼數?

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51, 52, 53, 54, 55, 56, 57, 58,

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# 數怎麼數?

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51, 52, 53, 54, 55, 56, 57, 58, 59,

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# 數怎麼數?

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# 數怎麼數?

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,  
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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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63, 64, 65,

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# 數怎麼數?

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# 數怎麼數？

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,  
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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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# 數怎麼數?

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16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,  
28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39,  
40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50,  
51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62,  
63, 64, 65, 66, 67, 68, 69, 70, 71, 72, ...,  $\infty$

▶ skip

# 命題

每個**整數**至少有一個有趣的故事 (性質)。

# 大綱

- 1 前言
- 2 數的演化
- 3 數論
- 4 自然數
- 5 命題證明
- 6 附錄

# 前言

## 例 1 (正常的數)

一般人看到計程車車牌為 1729, 會說那是個很冷門的數字, 但數學家卻認為 1729 是個有趣的數字。

## 例 1 (正常的數)

一般人看到計程車車牌為 1729, 會說那是個很冷門的數字, 但數學家卻認為 1729 是個有趣的數字。

### Story

1729 是可以表示成 2 個「立方和」**最小**的數。

$$1729 = 1^3 + 12^3$$

$$1729 = 9^3 + 10^3$$

$$1729 = 19 \times 91$$

## 例 2 (大數)

大數要怎麼數？

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大數要怎麼數？譬如：1 莫耳 (mole) =  $6.02 \times 10^{23}$

### Question

1 莫耳的數量需要多久才數得完？



## 例 2 (大數)

大數要怎麼數？譬如：1 莫耳 (mole) =  $6.02 \times 10^{23}$

### Question

1 莫耳的數量需要多久才數得完？

每秒數 100 次, (夠多了吧!)

100 / 秒

## 例 2 (大數)

大數要怎麼數？譬如：1 莫耳 (mole) =  $6.02 \times 10^{23}$

### Question

1 莫耳的數量需要多久才數得完？

每天有  $24 \times 60 \times 60 = 86,400$  秒，

$100 \times 86,400 = 8,640,000$  /天

## 例 2 (大數)

大數要怎麼數？譬如：1 莫耳 (mole) =  $6.02 \times 10^{23}$

### Question

1 莫耳的數量需要多久才數得完？

每年有 365 天，

$100 \times 86,400 \times 365 = 3,153,600,000$  /年

## 例 2 (大數)

大數要怎麼數？譬如：1 莫耳 (mole) =  $6.02 \times 10^{23}$

### Question

1 莫耳的數量需要多久才數得完？

地球年齡約 46 億年，

$$100 \times 86,400 \times 365 \times 4.6 \times 10^9 \approx 1.45 \times 10^{19}$$

## 例 2 (大數)

大數要怎麼數？譬如：1 莫耳 (mole) =  $6.02 \times 10^{23}$

### Question

1 莫耳的數量需要多久才數得完？

盤谷開天數到現在共數了：

$$100 \times 86,400 \times 365 \times 4.6 \times 10^9 \approx 1.45 \times 10^{19}$$

⇒ 還差 40,000 倍。

### Fact

一直還沒數完，我們都數不完，我們的兒子、孫子、孫孫子、孫孫孫...子都數不完。

# 例 3.14 (常數、長數、非常數)

## Definition

$$\pi = 2 \int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

- 無窮位數 (無理數)
- $\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$
- $\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{5}{5} \cdot \frac{6}{7} \cdot \frac{8}{7} \cdot \frac{8}{9} \dots$
- 圓周率日
  - 圓周率日: 3 月 14 日 (1:59, 15:9)
  - 終極圓周率日: 3/14/1592 6:54
  - 圓周近似值日: 7 月 22 日 (英國記為 22/7)

# 例 3.1415926535 (常數、長數、非常數)

$\pi = 3.1415926535897932384626433832795028841971693993$   
 $75105820974944592307816406286208998628034825342117$

## 口訣

山巔一寺一壺酒，爾樂苦煞吾，把酒吃，酒殺爾，殺不死，樂而樂，...

### • $\pi$ 的金氏世界紀錄

- 2002 年 12 月 6 日東京大學金田康正教授
- 用 600 小時，計算到小數點後 1 兆 2 千億位

# 例 $\infty$ (無窮大)

- 無窮大到底有多大？
  - 1 莫耳 vs. 無窮大
  - 無窮大+1 vs. 無窮大
  - 無窮大 $\times$ 2 vs. 無窮大
- 悖論
  - 龜兔賽跑
- 不可碰觸的無窮大
- 特殊的旅館《星際送奶者, 沉默者依翁》
  - 一個旅者
  - 無窮多個旅者



# 數的演化

# 數的由來

- 無數的世界: 比一比
- 因需要, 所以有數
- 進位
  - 2 進位: 多啦 A 夢、電腦
    - 優點: 不用背 99 乘法表。
    - 缺點: 100 要表示成  $110010_{(2)}$ ,  $0.4 = 0.\overline{0110}_{(2)}$ .
  - 8 進位: ET
  - 10 進位: 人
  - 其他: 12、24、60 進位
- 可以數的數、不可以數的數

# 數的層級

- 數數: 幼稚園
  - 日常生活
  - 數大便是美 ( $1 \text{ mole} = 6.02 \times 10^{23}$ )
  - 十進位
- 算術: 國小
- 數學: 國中、高中、大學
- 哲學: 大二以上數學系、人生課題

# 數論

# 數系

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathfrak{R} \subset \mathbb{C}$$

## • 實數 $\mathfrak{R}$

- 自然數:  $\{x \in \mathbb{N} \mid x = 1, 2, 3, \dots\}$
- 整數:  $\{x \in \mathbb{Z} \mid x = \dots, -2, -1, 0, 1, 2, \dots\}$
- 有理數:  $\{x \in \mathbb{Q} \mid x = \frac{m}{n} : m \in \mathbb{Z}, n \in \mathbb{Z}, n \neq 0\}$
- 無理數:  $\{x \mid \forall x \in \mathfrak{R}, x \notin \mathbb{Q}\}$

## • 虛數

- 複數  $\mathbb{C}$
- 代數數  $\mathbb{A}$  或  $\overline{\mathbb{Q}}$
- 超越數

# 自然數

- $1, 2, 3, \dots, \infty$  (無窮多個)
- 起源: **Peano's axioms**
- 需求: 記、計數
- 進位
- 阿拉伯數字

[◀ back](#)

# 自然數

- $1, 2, 3, \dots, \infty$  (無窮多個)
- 起源: **Peano's axioms**
- 需求: 記、計數
- 進位
- 阿拉伯數字
  - 773 年, 印度使節到巴格達
  - 阿爾·花刺子模 (Mohammad Al Khwarizmi)
  - algorithm

# 整數

- $\dots, -2, -1, 0, 1, 2, \dots$  (無窮多個)
- 需求:  $5 - 9$
- 0 是最後出現的數字
- 比「沒有」還小的數?
- 西方 18 世紀後才被普遍接受



# 整數

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- 西方 18 世紀後才被普遍接受

## Question

無窮多個自然數與無窮多個整數,何者個數較多?

# 有理數

- $\frac{m}{n}$ :  $\dots, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \dots$
- 需求:  $8x = 5$
- 富翁的遺產: 富翁共有 17 頭牛, 遺囑交代給大兒子  $\frac{1}{2}$ , 二兒子  $\frac{1}{3}$ , 小兒子  $\frac{1}{9}$ , 要怎麼分財產?

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## Question

無窮多個自然數與無窮多個有理數, 何者個數較多?

◀ back

# 無理數

- $x^2 = 2, \pi, e, \sqrt{2}$
- $\sqrt{2}$  殺死人

伊伯索 (Hippasus) 發現無理數嚇壞了畢達哥拉斯學派的人, 因此他們決定將此事祕而不宣, 並且把發現這件事的伊伯索給殺了。後來柏拉圖才幫我們整理出這個概念。

# 無理數

- $x^2 = 2$ ,  $\pi$ ,  $e$ ,  $\sqrt{2}$
- $\sqrt{2}$  殺死人

伊伯索 (Hippasus) 發現無理數嚇壞了畢達哥拉斯學派的人, 因此他們決定將此事祕而不宣, 並且把發現這件事的伊伯索給殺了。後來柏拉圖才幫我們整理出這個概念。

## Question

無窮多個自然數與無窮多個無理數, 何者個數較多?

# 實數

完備性有很多的說法,最主要的一種說法,最合乎直覺的,解析幾何的數與點的對應,將所有的有理點點在數線上,此外還有很多的點叫無理點,還要將它們加入,這樣才會得到  $\mathcal{R}$ ,也就是說,有理點在數線上雖然密密麻麻,但不構成全部的數線,必須把「漏洞」補起來,才會得到  $\mathcal{R}$  進一步的數學。

- 連續性: 沒有缺洞
- 完備性: 有上界遞增的實數數列必收斂至一個實數

## 0

- 沿革
  - ① 最後出現的數字
  - ② 最中間的整數
  - ③ 0 可被 2 整除, 所以是偶數
- $0 \times x = 0, \forall x \in \mathfrak{R}.$
- $x^0 = 1, \forall x \in \mathfrak{R}, (0^0 = 1 \text{ 定義})$
- $\frac{0}{0}$  無意義, 唯一沒有倒數的整數
- $0 + 0 = 0 \times 0 = 0$
- false

# 自然數



## 1

## • 沿革

- ① 第 1 個出現的數字
  - ② 第 1 個正整數, 自然數中最小的數, 最大的呢?
  - ③ 第 1 個奇數
  - ④ 第 1 個完全平方數
  - ⑤ 非質數亦非合數
- $1 \times x = x, \forall x \in \mathfrak{R}.$
  - $x^0 = 1, \forall x \in \mathfrak{R}$
  - $0.\bar{9}$  與 1 何者大?
  - 邊長 1 的正方形, 其對角線確確實實的長度是?
  - true

## 2

## • 沿革

- 1 第 1 個偶數
  - 2 第 2 個正整數
  - 3 第 1 個 (最小的) 質數?
  - 4 唯一的偶質數
- $x^2 + y^2 = z^2$ . (畢氏定理)
  - $x^n + y^n = z^n$ , 當  $n > 2$  時無整數解。(費瑪最後定理)
  - $2 + 2 = 2 \times 2$  (除了 0 外)
  - $e = 2.71828182845904523536 \dots$

## 3

- 第 3 個正整數
- 4 大千古奇案—幾何尺規做圖問題, 3 個無解題:
  - ① 化圓為方: 求作一正方形使其面積等於一已知圓;
  - ② 三等分任意角;
  - ③ 倍立方: 求作一立方體使其體積是一已知立方體的二倍。
- 沙灘上賣冰淇淋, 3 個攤位時無解。
- $x^n + y^n = z^n$ , 當  $n \geq 3$  時無整數解。(費瑪最後定理)

## 4

- $4 = 2^2$
- 除了 1 以外最小的平方數
- 四大千古奇案—幾何尺規做圖問題
  - ① 化圓為方: 求作一正方形使其面積等於一已知圓;
  - ② 三等分任意角;
  - ③ 倍立方: 求作一立方體使其體積是一已知立方體的二倍。
  - ④ 做正十七邊形。

## 5

- 5 進位: 算盤、羅馬數字、正、手、貨幣
- 5 線譜、5 個母音、5 行、古代 5 音階
- 黃金分割數

$$\frac{1 + \sqrt{5}}{2}$$

- 第 1 個奇數
- 第 1 個質數

## 6

- 完全數  $6 = 1 \times 2 \times 3 = 1 + 2 + 3$
- 3 角形數  $\{1, 3, 6, 10\}$
- 正 6 邊形最長的對角線就等於其內接圓的直徑
- 能夠密鋪平面的正多邊形只有正三角形, 正方形與正六邊形三種情形, 再沒有其他的了。這是三角形三內角和為  $180^\circ$  的簡單推論。

# 命題證明

# 證明的方法

- 演譯法
- 歸納法
- 反證法
- 其他

[◀ back](#)

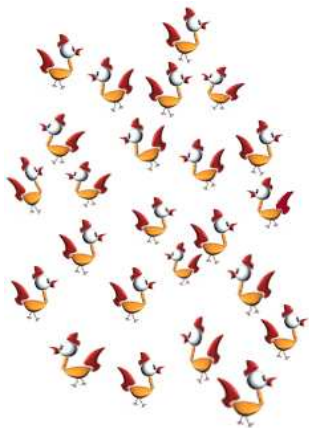


# 證明

# The End

# 比一比

在無數的世界, 下圖有「多少」隻雞?



# 比一比

共有「這麼」多隻雞。



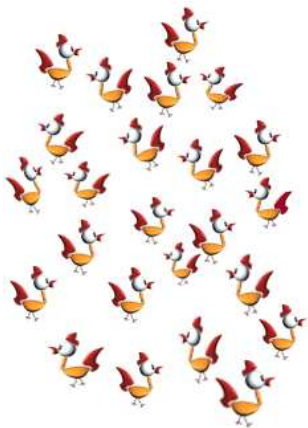
# 比一比

每隻雞發一顆石頭，



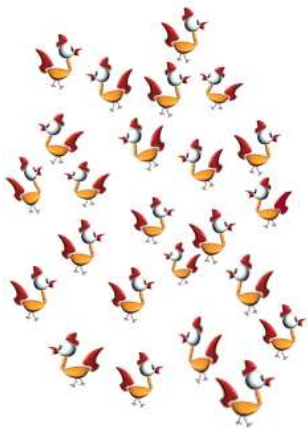
# 比一比

拿出來，



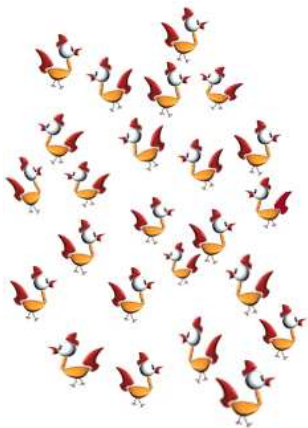
# 比一比

收集起來，



# 比一比

瞧! 總共就這麼多。





# 比一比

沒人知道多少, 就你看到的「這麼」多。



# PEANO'S AXIOMS

Peano's axioms (5 公設)  $\mathbb{N} = \{1, 2, 3, \dots\}$

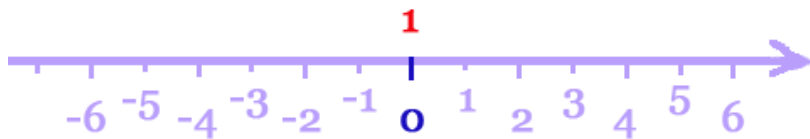
- 1 為自然數
- 任何自然數都有後繼元素 ( $n \rightarrow n'$ )
- 兩個後繼元素相同, 則兩數相同 (等)  $m' = n' \Rightarrow m = n$
- 1 不為任何自然數的後繼元素
- If  $A \subset \mathbb{N}$  satisfies both
  - $1 \in A$
  - $n \in A$  implies  $n' \in A$

then  $A = \mathbb{N}$

# 整數與自然數何者較多



# 整數與自然數何者較多



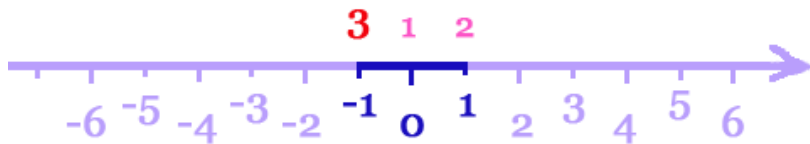
$$\{1\} \rightarrow \{0\}$$

# 整數與自然數何者較多



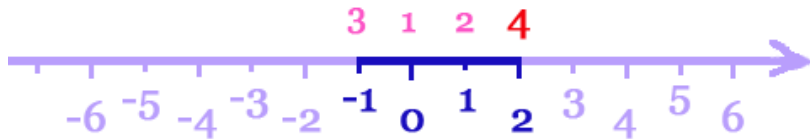
$$\{2\} \rightarrow \{1\}$$

# 整數與自然數何者較多



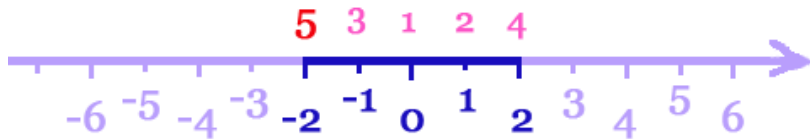
$$\{3\} \rightarrow \{-1\}$$

# 整數與自然數何者較多



$$\{4\} \rightarrow \{2\}$$

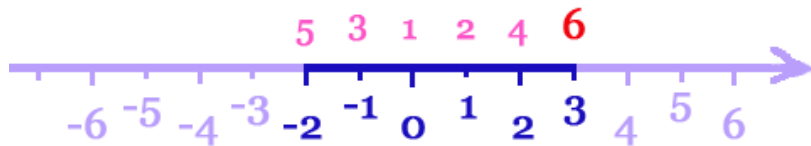
# 整數與自然數何者較多



$$\{5\} \rightarrow \{-2\}$$

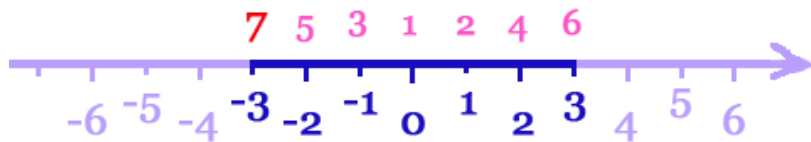


# 整數與自然數何者較多



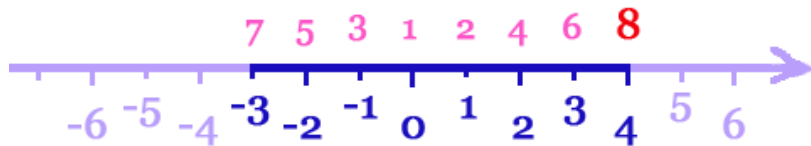
$$\{6\} \rightarrow \{3\}$$

# 整數與自然數何者較多



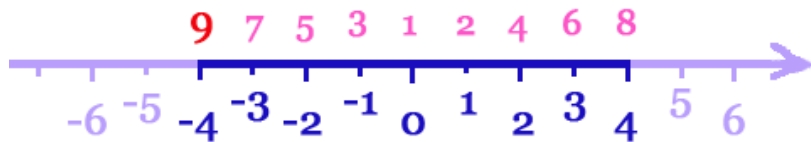
$$\{7\} \rightarrow \{-3\}$$

# 整數與自然數何者較多



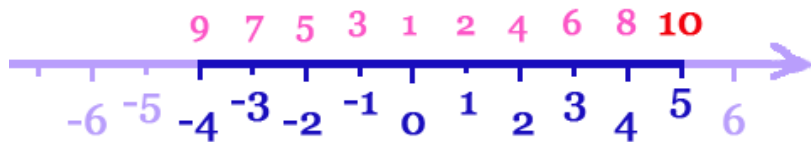
$$\{8\} \rightarrow \{4\}$$

# 整數與自然數何者較多



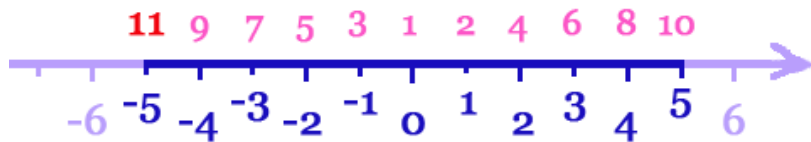
$$\{9\} \rightarrow \{-4\}$$

# 整數與自然數何者較多



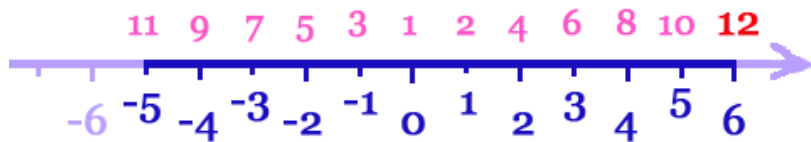
$$\{10\} \rightarrow \{5\}$$

# 整數與自然數何者較多



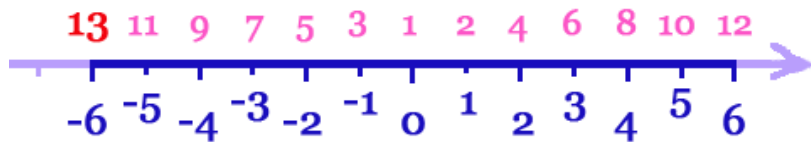
$$\{11\} \rightarrow \{-5\}$$

# 整數與自然數何者較多



$$\{12\} \rightarrow \{6\}$$

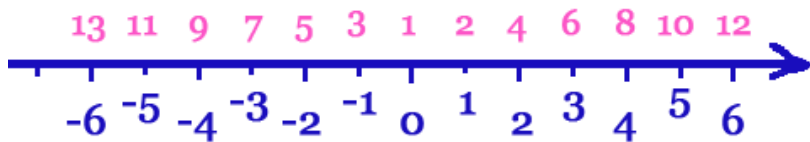
# 整數與自然數何者較多



$$\{13\} \rightarrow \{-6\}$$



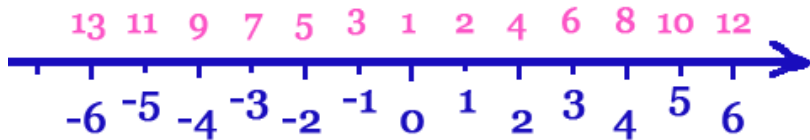
# 整數與自然數何者較多



$$n \rightarrow \begin{cases} \frac{n}{2} & \text{若 } n \in \text{偶數,} \\ -\frac{n-1}{2} & \text{若 } n \in \text{奇數.} \end{cases}$$

1 個應對 1 個, 剛剛好, 也不多也不少。

# 整數與自然數何者較多



$$n \rightarrow \begin{cases} \frac{n}{2} & \text{若 } n \in \text{偶數,} \\ -\frac{n-1}{2} & \text{若 } n \in \text{奇數.} \end{cases}$$

1 個應對 1 個, 剛剛好, 也不多也不少。

## Fact

整數與自然數一樣多。

# 分遺產

## Question

富翁的遺產: 富翁共有 17 頭牛, 遺囑交代給大兒子  $1/2$ , 二兒子  $1/3$ , 小兒子  $1/9$ , 要怎麼分財產?

## Answer

兒子們去找村長, 村長把自己的 1 頭牛牽來, 湊成 18 頭,  
大兒子:  $18 \times 1/2 = 9$ ,  
二兒子:  $18 \times 1/3 = 6$ ,  
小兒子:  $18 \times 1/9 = 2$ ,  
剩 1 頭牛, 村長又把牛牽回家。

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	...
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	...
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	...
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	...
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	...
⋮	⋮	⋮	⋮	⋮	⋮

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<b>1</b> (1)	(1,2)	(1,3)	(1,4)	(1,5)	...
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	...
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	...
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	...
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	...
⋮	⋮	⋮	⋮	⋮	⋮

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<b>(1,1)</b>	<b>(1,2)</b>	(1,3)	(1,4)	(1,5)	...
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	...
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	...
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	...
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	...
⋮	⋮	⋮	⋮	⋮	⋮

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<b>(1,1)</b>	<b>(1,2)</b>	(1,3)	(1,4)	(1,5)	...
<b>(2,1)</b>	<del>(2,2)</del>	(2,3)	(2,4)	(2,5)	...
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	...
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	...
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	...
⋮	⋮	⋮	⋮	⋮	⋮

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<del>(1,1)</del>	<del>(1,2)</del>	<del>(1,3)</del>	(1,4)	(1,5)	...
<del>(2,1)</del>	<del>(2,2)</del>	(2,3)	(2,4)	(2,5)	...
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	...
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	...
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	...
⋮	⋮	⋮	⋮	⋮	⋮



# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<b>(1,1)</b>	<b>(1,2)</b>	<b>(1,3)</b>	(1,4)	(1,5)	...
<b>(2,1)</b>	<del>(2,2)</del>	<b>(2,3)</b>	(2,4)	(2,5)	...
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	...
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	...
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	...
⋮	⋮	⋮	⋮	⋮	⋮

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<del>(1,1)</del>	<del>(1,2)</del>	<del>(1,3)</del>	(1,4)	(1,5)	...
<del>(2,1)</del>	<del>(2,2)</del>	<del>(2,3)</del>	(2,4)	(2,5)	...
(3,1)	<del>(3,2)</del>	<del>(3,3)</del>	(3,4)	(3,5)	...
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	...
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	...
⋮	⋮	⋮	⋮	⋮	⋮

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<del>(1,1)</del>	<del>(1,2)</del>	<del>(1,3)</del>	(1,4)	(1,5)	...
<del>(2,1)</del>	<del>(2,2)</del>	<del>(2,3)</del>	(2,4)	(2,5)	...
<del>(3,1)</del>	<del>(3,2)</del>	<del>(3,3)</del>	(3,4)	(3,5)	...
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	...
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	...
⋮	⋮	⋮	⋮	⋮	⋮

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<del>(1,1)</del>	<del>(1,2)</del>	<del>(1,3)</del>	<del>(1,4)</del>	(1,5)	...
(2,1)	<del>(2,2)</del>	(2,3)	(2,4)	(2,5)	...
<del>(3,1)</del>	<del>(3,2)</del>	<del>(3,3)</del>	(3,4)	(3,5)	...
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	...
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	...
⋮	⋮	⋮	⋮	⋮	⋮

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<del>(1,1)</del>	<del>(1,2)</del>	<del>(1,3)</del>	<del>(1,4)</del>	(1,5)	...
<del>(2,1)</del>	<del>(2,2)</del>	(2,3)	<del>(2,4)</del>	(2,5)	...
<del>(3,1)</del>	<del>(3,2)</del>	<del>(3,3)</del>	<del>(3,4)</del>	(3,5)	...
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	...
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	...
⋮	⋮	⋮	⋮	⋮	⋮

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<del>(1,1)</del>	<del>(1,2)</del>	<del>(1,3)</del>	<del>(1,4)</del>	(1,5)	...
<del>(2,1)</del>	<del>(2,2)</del>	<del>(2,3)</del>	<del>(2,4)</del>	(2,5)	...
<del>(3,1)</del>	<del>(3,2)</del>	<del>(3,3)</del>	<del>(3,4)</del>	(3,5)	...
(4,1)	(4,2)	<del>(4,3)</del>	<del>(4,4)</del>	(4,5)	...
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	...
⋮	⋮	⋮	⋮	⋮	⋮

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<b>1</b>	<b>2</b>	<b>4</b>	<b>8</b>	(1,5) ...
<b>3</b>	<del>(2,2)</del>	<b>5</b>	<del>(2,4)</del>	(2,5) ...
<b>7</b>	<del>(3,2)</del>	<del>(3,3)</del>	<b>9</b>	(3,5) ...
<b>11</b>	<del>(4,2)</del>	<b>10</b>	<del>(4,4)</del>	(4,5) ...
(5,1)	(5,2)	(5,3)	(5,4)	(5,5) ...
⋮	⋮	⋮	⋮	⋮

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<b>1</b>	<b>2</b>	<b>4</b>	<b>8</b>	<b>12</b>	...
<del>(1,1)</del>	<del>(1,2)</del>	<del>(1,3)</del>	<del>(1,4)</del>	<del>(1,5)</del>	...
<b>3</b>	<del>(2,2)</del>	<b>5</b>	<del>(2,4)</del>	(2,5)	...
<del>(2,1)</del>	<del>(3,2)</del>	<del>(3,3)</del>	<del>(3,4)</del>	(3,5)	...
<b>7</b>	<del>(4,2)</del>	<b>10</b>	<del>(4,4)</del>	(4,5)	...
<del>(4,1)</del>	<del>(5,2)</del>	(5,3)	(5,4)	(5,5)	...
<b>11</b>					...
<del>(5,1)</del>					...
⋮	⋮	⋮	⋮	⋮	⋮



# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<del>(1,1)</del>	<del>(1,2)</del>	<del>(1,3)</del>	<del>(1,4)</del>	<del>(1,5)</del>	...
<del>(2,1)</del>	<del>(2,2)</del>	<del>(2,3)</del>	<del>(2,4)</del>	<del>(2,5)</del>	...
<del>(3,1)</del>	<del>(3,2)</del>	<del>(3,3)</del>	<del>(3,4)</del>	<del>(3,5)</del>	...
<del>(4,1)</del>	<del>(4,2)</del>	<del>(4,3)</del>	<del>(4,4)</del>	<del>(4,5)</del>	...
<del>(5,1)</del>	<del>(5,2)</del>	<del>(5,3)</del>	<del>(5,4)</del>	<del>(5,5)</del>	...
⋮	⋮	⋮	⋮	⋮	⋮

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<del>(1,1)</del>	<del>(1,2)</del>	<del>(1,3)</del>	<del>(1,4)</del>	<del>(1,5)</del>	...
<del>(2,1)</del>	<del>(2,2)</del>	<del>(2,3)</del>	<del>(2,4)</del>	<del>(2,5)</del>	...
<del>(3,1)</del>	<del>(3,2)</del>	<del>(3,3)</del>	<del>(3,4)</del>	<del>(3,5)</del>	...
<del>(4,1)</del>	<del>(4,2)</del>	<del>(4,3)</del>	<del>(4,4)</del>	<del>(4,5)</del>	...
⋮	⋮	⋮	⋮	⋮	↘

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<del>(1,1)</del>	<del>(1,2)</del>	<del>(1,3)</del>	<del>(1,4)</del>	<del>(1,5)</del>	...
<del>(2,1)</del>	<del>(2,2)</del>	<del>(2,3)</del>	<del>(2,4)</del>	<del>(2,5)</del>	...
<del>(3,1)</del>	<del>(3,2)</del>	<del>(3,3)</del>	<del>(3,4)</del>	<del>(3,5)</del>	...
<del>(4,1)</del>	<del>(4,2)</del>	<del>(4,3)</del>	<del>(4,4)</del>	<del>(4,5)</del>	...
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	...
⋮	⋮	⋮	⋮	⋮	⋮

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<b>1</b>	<b>2</b>	<b>4</b>	<b>8</b>	<b>12</b>	...
<del>(1,1)</del>	<del>(1,2)</del>	<del>(1,3)</del>	<del>(1,4)</del>	<b>13</b>	...
<b>3</b>	<del>(2,2)</del>	<b>5</b>	<del>(2,4)</del>	<del>12,5</del>	...
<del>(2,1)</del>	<del>(3,2)</del>	<del>(3,3)</del>	<del>(3,4)</del>	<b>14</b>	...
<b>7</b>	<del>(4,2)</del>	<del>(4,3)</del>	<del>(4,4)</del>	<b>15</b>	...
<del>(3,1)</del>	<del>(4,1)</del>	<del>(4,1)</del>	<del>(4,1)</del>	<del>(4,1)</del>	...
<b>11</b>	<del>(5,2)</del>	<del>(5,3)</del>	<b>16</b>	<del>(5,5)</del>	...
<del>(4,1)</del>	<del>(5,1)</del>	<del>(5,1)</del>	<del>(5,1)</del>	<del>(5,1)</del>	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<b>1</b>	<b>2</b>	<b>4</b>	<b>8</b>	<b>12</b>	...
<del>(1,1)</del>	<del>(1,2)</del>	<del>(1,3)</del>	<del>(1,4)</del>	<b>13</b>	...
<b>3</b>	<del>(2,2)</del>	<b>5</b>	<del>(2,4)</del>	<del>12,5</del>	...
<del>(2,1)</del>	<del>(3,2)</del>	<del>(3,3)</del>	<del>(3,4)</del>	<b>14</b>	...
<b>7</b>	<del>(4,2)</del>	<del>(4,3)</del>	<del>(4,4)</del>	<b>15</b>	...
<del>(3,1)</del>	<del>(5,2)</del>	<del>(5,3)</del>	<del>(5,4)</del>	<del>(5,5)</del>	...
<b>11</b>	<del>(5,1)</del>	<b>17</b>	<b>16</b>	<del>(5,5)</del>	...
<del>(4,1)</del>	<del>(5,1)</del>	<del>(5,2)</del>	<del>(5,3)</del>	<del>(5,4)</del>	...
⋮	⋮	⋮	⋮	⋮	↘

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<b>1</b>	<b>2</b>	<b>4</b>	<b>8</b>	<b>12</b>	...
<del>(1,1)</del>	<del>(1,2)</del>	<del>(1,3)</del>	<del>(1,4)</del>	<b>13</b>	...
<b>3</b>	<del>(2,2)</del>	<b>5</b>	<del>(2,4)</del>	<del>12,5</del>	...
<del>(2,1)</del>	<del>(3,2)</del>	<del>(3,3)</del>	<del>(3,4)</del>	<b>14</b>	...
<b>7</b>	<del>(4,2)</del>	<del>(4,3)</del>	<del>(4,4)</del>	<b>15</b>	...
<del>(4,1)</del>	<del>(5,2)</del>	<del>(5,3)</del>	<del>(5,4)</del>	<del>(5,5)</del>	...
<b>11</b>	<b>18</b>	<b>17</b>	<b>16</b>		...
<del>(5,1)</del>	<del>(5,2)</del>	<del>(5,3)</del>	<del>(5,4)</del>	<del>(5,5)</del>	...
⋮	⋮	⋮	⋮	⋮	↘

# 有理數與自然數何者較多

將  $\frac{m}{n}$  表示成  $(m, n)$ , 如此一來, 就可將所有的有理數「列」出來,

<del>(1,1)</del>	<del>(1,2)</del>	<del>(1,3)</del>	<del>(1,4)</del>	<del>(1,5)</del>	...
<del>(2,1)</del>	<del>(2,2)</del>	<del>(2,3)</del>	<del>(2,4)</del>	<del>(2,5)</del>	...
<del>(3,1)</del>	<del>(3,2)</del>	<del>(3,3)</del>	<del>(3,4)</del>	<del>(3,5)</del>	...
<del>(4,1)</del>	<del>(4,2)</del>	<del>(4,3)</del>	<del>(4,4)</del>	<del>(4,5)</del>	...
<del>(5,1)</del>	<del>(5,2)</del>	<del>(5,3)</del>	<del>(5,4)</del>	<del>(5,5)</del>	...
⋮	⋮	⋮	⋮	⋮	⋮

## Fact

有理數與自然數一樣多, 也與整數一樣多。

# 無理數與自然數何者較多

假設  $(0, 1)$  為可列,  $x_1, x_2, x_3, \dots$ ,



# 無理數與自然數何者較多

假設  $(0, 1)$  為可列,  $x_1, x_2, x_3, \dots$ ,

$$x_1 = 0.a_{11}a_{12}a_{13}a_{14}a_{15} \cdots$$

$$x_2 = 0.a_{21}a_{22}a_{23}a_{24}a_{25} \cdots$$

$$x_3 = 0.a_{31}a_{32}a_{33}a_{34}a_{35} \cdots$$

⋮

# 無理數與自然數何者較多

假設  $(0, 1)$  為可列,  $x_1, x_2, x_3, \dots$ ,

$$x_1 = 0.a_{11}a_{12}a_{13}a_{14}a_{15} \cdots$$

$$x_2 = 0.a_{21}a_{22}a_{23}a_{24}a_{25} \cdots$$

$$x_3 = 0.a_{31}a_{32}a_{33}a_{34}a_{35} \cdots$$

⋮

造一個新數  $x = 0.b_1b_2b_3b_4 \cdots$ ,  $b_k \neq a_{kk}$ ,  $\forall k \in \mathbb{N}$ ,

# 無理數與自然數何者較多

假設  $(0, 1)$  為可列,  $x_1, x_2, x_3, \dots$ ,

$$x_1 = 0.a_{11}a_{12}a_{13}a_{14}a_{15} \cdots$$

$$x_2 = 0.a_{21}a_{22}a_{23}a_{24}a_{25} \cdots$$

$$x_3 = 0.a_{31}a_{32}a_{33}a_{34}a_{35} \cdots$$

⋮

造一個新數  $x = 0.b_1b_2b_3b_4 \cdots$ ,  $b_k \neq a_{kk}, \forall k \in \mathbb{N}$ ,

$x \in (0, 1)$ , 但  $x$  皆不為  $x_1, x_2, x_3, \dots$ ,  $\Rightarrow$  矛盾,

所以  $(0, 1)$  不可列, 即無理數不可列。

# 無理數與自然數何者較多

假設  $(0, 1)$  為可列,  $x_1, x_2, x_3, \dots$ ,

$$x_1 = 0.a_{11}a_{12}a_{13}a_{14}a_{15} \cdots$$

$$x_2 = 0.a_{21}a_{22}a_{23}a_{24}a_{25} \cdots$$

$$x_3 = 0.a_{31}a_{32}a_{33}a_{34}a_{35} \cdots$$

⋮

造一個新數  $x = 0.b_1b_2b_3b_4 \cdots$ ,  $b_k \neq a_{kk}$ ,  $\forall k \in \mathbb{N}$ ,

$x \in (0, 1)$ , 但  $x$  皆不為  $x_1, x_2, x_3, \dots$ ,  $\Rightarrow$  矛盾,

所以  $(0, 1)$  不可列, 即無理數不可列。

## Fact

無理數比自然數或整數多很多, 也比自然數加上整數多。

# 奇數 $2n + 1$

- 無法被 2 除的盡的數
  - $1, 3, 5, \dots, 2n + 1$
  - 奇 + 奇 = 偶
  - 奇 + 偶 = 奇
  - 奇  $\times$  奇 = 奇

$$(2n + 1)(2n + 1) = 4n^2 + 4n + 1$$

- 

[◀ back](#)

# 偶數 $2n$

- 被 2 除的盡的數
  - $2, 4, 6, \dots, 2n$
  - 偶 + 偶 = 偶
  - 偶  $\times$  偶 = 偶
- Adrian Monk
- 畢達哥拉斯

[◀ back](#)

# 質數

算數基本定理的唯一性 (存在唯一質因數分解)

- 1 不為質數, 它破壞了算數基本定理的唯一性 (存在唯一質因數分解)。
- 找質數的方法
  - 歐幾里德: 質數有無限個
  - 梅森尼質數:  $2^n - 1$  (最大的質數:  $2^{32582657} - 1$ )
  - 費瑪:  $2^{2^n} + 1, \{5, 7, 257, 65537\}$
  - 歐拉:  $n^2 - n + 41, \{n = 1, \dots, 40\}$
  - 1742 年郭德巴哈: 大於 4 的偶數, 皆可表示成兩個奇質數和 (2006 年 7 月, 檢驗了  $4 \times 10^{17}$  以內都對)
- 17 年蟬

# 畢氏定理

畢達哥拉斯百牛定理畢達哥拉斯更說:「萬物皆是數。」數學是等於哲學的

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[◀ back](#)



# 費瑪最後定理 (FERMAT'S LAST THEOREM)

## 1

皮埃爾·德·費瑪 (Pierre de Fermat) 生平:

- 法國人 1601–1665,
- 正業是律師、宮廷顧問。
- 被數學史家貝爾尊稱為「業餘數學家之王」。
- 在解析幾何、數論、無窮小分析 (微積分之前身) 和概率論方面, 他都有重要之貢獻。

## 費瑪最後定理 2

關於定理:

- 古希臘數學家丟番圖 (Diophantus) 的著作《算術》(Arithmetica)
- 費瑪的壞習慣 (費瑪 48 個猜想)
- 「我有一個對這個命題十分美妙的證明, 但是因為這裡空白太小了, 我無法寫下」(1637)
- Euler, Gauss, Germain, Dirichlet, Legendre, Cauchy) . . . , 無一莫不折腰於「費瑪最後定理」之前。
- 此定理高懸 350 年。

# 費瑪最後定理 3

- 軼聞
  - 救了德國數學家 P. Wolfshehl
  - Wolfshehl 於 1908 年遺贈 10 萬馬克懸賞
- 安德魯懷爾斯證明出費瑪最後定理:
  - 安德魯懷爾斯 (Andrew Wiles) 10 歲就立志於此
  - 於 1993 年發表「費瑪最後定理」的證明, 但
  - 終於 1995 年 5 月將其研究發表在《數學年刊》上

# 費瑪最後定理 4

## 評論

- 懷爾斯花費 7 年時間以 130 頁的篇幅證明。
- 20 世紀的現代數學工具而非費瑪時代的古典數學。
- 130 頁的證明似乎不是費瑪所稱「美妙」的證明。
- 結束了 350 年數學家的努力。
- 錯誤? 惡作劇? 玩笑?

[◀ back](#)

# 完全數

上帝利用 6 天的時間創造了世界; 月亮繞行地球只須 28 天

- 它們等於它本身之外全部因數的和
- 它們都能寫成連續自然數之和
- 它們的全部因數的倒數之和都是 2

$$6 = 1 + 2 + 3$$

$$28 = 1 + 2 + 3 + 4 + 5 + 6 + 7 = 1 + 2 + 4 + 7 + 14$$